Packet Error Probabilities in Direct Sequence Spread Spectrum Packet Radio Networks with BCH Codes

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Abstract

In this paper, we compute an upper bound on the packet error probability induced in direct sequence spread spectrum networks, when BCH codes are used for the encoding of the packets. The bound, which we introduce, is valid independently of whether signals arrive with equal or unequal powers at the receiver site. Furthermore, it has a simple form and it is easy to compute. In addition to that, it is valid for other classes of forward error correction codes (e.g. convolutional codes), but in this paper numerical results are presented for BCH codes only.

1. Introduction

The problem of computing packet error probabilities in direct sequence spread spectrum packet radio networks is difficult. Packet errors are caused by a combination of noise at the receivers and interference between packet transmissions which overlap in time. The interference between packet transmissions produces dependent errors at the output of the demodulator. A lot of work has been directed towards the evaluation of the bit error probability in direct sequence spread spectrum networks ([1], [2]). The dependency of the bit errors does not allow us to extend the results in [1] and [2], in order to compute the packet error probability.

To the best of our knowledge, the first serious effort to compute packet error probabilities in direct sequence spread spectrum networks was conducted in [4]. In [4] the authors compute an upper bound on the packet error probability induced in a direct sequence spread spectrum packet radio network, which utilizes binary convolutional coding, hard-decision demodulaton Viterbi decoding and random signature sequences.

The upper bound on the packet error probability, derived in [4], had been proven to be valid, only when the signals arrive with equal power at the receiver site. This is a severe limitation, because, in general, signals arrive at the receiver site with unequal powers. In this paper, we present an upper bound on the packet error probability induced in direct sequence spread spectrum packet radio networks, when BCH codes are used for the encoding of the packets. Our bound is valid independently of whether signals arrive with equal or unequal powers at the receiver site. The upper bound, which we introduce, has a simple form and it is easy to compute. In addition to that, it is valid for other classes of codes (e.g. convolutional codes), but in this paper numerical results are presented for BCH codes only.

2. Model-Preliminaries

The model for direct sequence spread spectrum transmission considered here is described in [5]. The only difference is that the signature sequence is assumed to be a sequence of independent, identically distributed, binary random variables, each equally likely to be +1 or -1. Each transmitter in the network has such a sequence, and each sequence is assumed to be independent of the sequences of other transmitters.

Let us now assume that we have a slotted channel (i.e. packet transmissions initiate at the beginnings of slots), $K(K>1)$ packet transmissions occur within a slot, and a receiver locks on to packet #1 (packets are indexed #1, #2, ..., #K). Each packet originates from a different transmitter in the network. A packet is exactly one codeword from an (M,L) BCH code (M=total number of codeword bits, L=total number of information bits; the bits of a codeword are indexed from 0 up to $M-1$. Our

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objective is to compute the probability that the receiver decodes packet #1 incorrectly.

The receiver is assumed to be a correlation receiver. The output of the receiver, corresponding to the mth bit $(0 \le m \le M-1)$ of packet #1, is the random variable (see [3] for more details)

$$Z_m = n_m + (2^{-1}P_1)^{1/2} T\{b_m^{(1)} + \sum_{i=2}^{K} (P_i/P_1)^{1/2} I_{i,1}^m(\underline{b}_i^m, \tau_i, \theta)\} \quad ; 0 \le m \le M-1 \qquad (1)$$

Each n_m is a Gaussian random variable with zero mean and variance $N_o T/4$, where $N_o/2$ is the two sided spectral density of the white Gaussian noise and T is the data bit duration. The random variables n_m $(0 \le m \le M-1)$ are independent. The random variable $b_m^{(1)}$ represents the mth bit of packet #1; its value is either +1 or -1. The vector $\underline{b}_i^m$ represents a pair of consecutive data bits of packet #i. In particular, $\underline{b}_i^m = (b_{m-1}^{(i)}, b_m^{(i)})$, and each data bit $b_m^{(i)}$ is either +1 or -1. Each τ_i or θ_i is a random variable representing the time delay (modulo T) or the phase angle (modulo 2π), respectively, of packet #i relative to packet #1. As in [3], we take the range or τ_i to be the interval $[0,T]$ and the range of θ_i to be the interval $[0,2\pi]$. Finally, P_i is the power of packet #i at the receiver.

The function $I_{i,1}^m$, which appears in (1) represents the normalized multiple access interference due to packet #i. This function is defined by

$$I_{i,1}^m(\underline{b}_i^m, \tau, \theta) = T^{-1}[b_{m-1}^{(i)} R_{i,1}^m(\tau) + b_m^{(i)} \hat{R}_{i,1}^m(\tau)]\cos\theta \qquad (2)$$

where the functions $R_{i,1}^m$ and $\hat{R}_{i,1}^m$ are given by

$$R_{i,1}^m(\tau) = \int_{mT}^{mT+\tau} a_i(t-\tau)a_1(t)dt \qquad (3)$$

$$\hat{R}_{i,1}^m(\tau) = \int_{mT+\tau}^{(m+1)T} a_i(t-\tau)a_1(t)dt \qquad (4)$$

Note that the $a_i(t)$ and the $a_1(t)$ in (3) and (4) are the spectral spreading signals corresponding to packets #i and #1, respectively. In fact,

$$a_i(t) = \sum_{j=-\infty}^{+\infty} a_j^{(i)} \psi(t-jT_c) \quad ; 1 \le i \le K \qquad (5)$$

where $\{a_j^{(i)}\}$ is the signature sequence corresponding to packet #i, $\psi(t)$ is the chip waveform, and T_c is the chip duration. In this paper, we assume a rectangular chip waveform. Hence,

$$\psi(t) = \begin{cases} 1 & 0 \le t \le T_c \\ 0 & \text{otherwise} \end{cases} \qquad (6)$$

The decoder decides that the mth bit of packet #1 is +1 or -1 if $Z_m > 0$ or $Z_m < 0$, respectively. It is easy to show that the mth bit of packet #1 is decoded correctly by the above decoder if and only if the random variable

$$X_m = n_m^* + [1 + \sum_{i=2}^{K} b_m^{(1)} (P_i/P_1)^{1/2} I_{i,1}^m \ (\underline{b}_i^m, \tau_i, \theta_i)] \quad ; 0 \le m \le M-1 \qquad (7)$$

is positive. In (7), each n_m^* is a Gaussian random variable with mean 0 and variance $N_o/2E_b$, where $E_b = P_1 T$ is the energy per data bit of packet #1. The random variables $n_m^* (0 \le m \le M-1)$ are statistically independent.

Let us now denote by S a random variable, which represents the number of random variables $X_m (0 \le m \le M-1)$ that are negative.

Then,

$$P_e(K) = \Pr(S > e) \qquad (8)$$

where e corresponds to the error correction capability of the BCH code. We will state two propostions, without proof (for more details see [7]).

Proposition 1. For the computation of $P_e(K)$ the τ_i's $(2 \le i \le K)$ need be known only to the nearest chip.

Proposition 2. $P_e(K)$ is independent of the values of the data bit sequences $\{b_m^{(i)}\}_{m=0}^{M-1}$ for $1 \le i \le K$.

An immediate consequence of propositions 1 and 2, is that the random variable X_m in (7) assumes the following equivalent form (see also (2) through (6))

$$X_m = n_m^* + 1 + \sum_{i=2}^{K} [(P_i/P_1)^{1/2} [a_{mN-1}^{(i)} a_{mN}^{(1)}]\tau_i/T_c + [a_{mN}^{(i)} a_{mN}^{(1)} + \ldots a_{mN+N-1}^{(i)} a_{mN+N-1}^{(1)}](1-\tau_i/T_c) + [a_{mN}^{(i)} a_{mN+1}^{(1)} + \ldots a_{mN+N-2}^{(i)} a_{mN+N-1}^{(1)}] \tau_i/T_c]\cos\theta_i/N \quad ; 0 \le m \le M-1 \qquad (9)$$

In (9) we assumed that each of the spread spectrum signals has N chips per bit. Let us make an important observation.

Observation 1. Given the phase (θ_i) and the delay (τ_i) of each transmission ($2 \le i \le K$), the random variables $X_m (0 \le m \le M-1)$ are not independent.

To prove our observation we show, in [7] that for N=2, K=3, $\theta_2=\theta_3=0$, $\tau_2=\tau_3=T_c/2$ and $P_2/P_1=P_3/P_1=1$ the following inequality is true.

$$\Pr(X_0<0 \cap X_1<0) \neq \Pr(X_0<0)\Pr(X_1<0) \quad (10)$$

In [7] we point out that for some examples of packet radio networks the random variables, X_m are conditionally independent given all delays and phases (e.g. when bit interleaving is used). Furthermore, it is the author's belief that the packet error probability $P_e(K)$ will not be severely affected if we treat the random variables X_m as conditionally independent, provided that $K<<N$. As a result, the derivation of the upper bound on the packet error probability $P_e(K)$, presented in the next section, will be based on the following proposition

Proposition 3. Given the phase (θ_i) and the delay (τ_i) of each transmission ($2 \le i \le K$), the random variables $X_m (0 \le m \le M-1)$ are independent.

3. An upper bound on the packet error probability.

3.a Derivaton of the upper bound

Let us define the random vectors

$$\underline{\theta} = (\theta_2 \theta_3 \ldots \theta_K) \quad (11)$$
$$\underline{\tau} = (\tau_2 \tau_3 \ldots \tau_K)$$

Let us denote by $f_{\underline{\tau},\underline{\theta}}(\hat{\tau},\hat{\theta})$ the joint probability density function of the random vectors $\underline{\tau}$ and $\underline{\theta}$. The first step in our effort to compute an upper bound on $P_e(K)$ (see (8) in section 2) is to condition on $\underline{\theta}$ (all phases) and $\underline{\tau}$ (all delays). Then, we get

$$P_e(K)=\int_{\hat{r}}\int_{\hat{\theta}} \Pr(S>e/\underline{\tau}=\hat{\underline{\tau}},\underline{\theta}=\hat{\underline{\theta}}) f_{\tau,\theta}(\hat{\underline{\tau}},\hat{\underline{\theta}})d\hat{\underline{\theta}}d\hat{\underline{\tau}} \quad (12)$$

Due to proposition 3 and formula (9), we can write

$$\Pr(S>e/\underline{\tau}=\hat{\underline{\tau}},\underline{\theta}=\hat{\underline{\theta}})=\sum_{i=e+1}^{M}\begin{bmatrix}M\\i\end{bmatrix}p^i(1-p)^{M-i}=g(p) \quad (13)$$

where

$$p=\Pr(X_0<0)=\Pr(n_0^*+1+\sum_{i=2}^{K}(P_i/P_1)^{1/2} I_i^0(\hat{\tau}_i,\hat{\theta}_i)<0) \quad (14)$$

with

$$I_i^0(\hat{\tau}_i,\hat{\theta}_i)=\{a_{-1}^{(i)}a_0^{(1)}\hat{\tau}_i/T_c+[a_0^{(i)}a_0^{(1)}+\ldots+a_{N-1}^{(i)}a_{N-1}^{(1)}](1-\hat{\tau}_i/T_c)+[a_0^{(i)}a_1^{(1)}+\ldots+a_{N-2}^{(i)}a_{N-1}^{(1)}]\hat{\tau}_i/T_c\}\cos\hat{\theta}_i/N \quad ;2 \le i \le K \quad (15)$$

The second step in our work is to find an upper bound p_u on the probability p, which is independent of $\hat{\underline{\tau}}$ and $\hat{\underline{\theta}}$. By doing so, we can upper bound g(p) (see [13]) by $g(p_u)$, since g(p) is an increasing function of p. As a result, we can write

$$P_e(K) \le g(p_u) \quad (16)$$

Let us start by defining the random variable Y such that

$$Y=n_0^*+\{\sum_{i=2}^{K}(P_i/P_1)^{1/2}\hat{I}_i^0(\hat{\tau}_i,\hat{\theta}_i)\}-1 \quad (17)$$

where

$$\hat{I}_i^0(\hat{\tau}_i,\hat{\theta}_i)=\{a_{-1}^{(i)}\hat{a}_0^{(1)}\hat{\tau}_i/T_c+[a_0^{(i)}\hat{a}_0^{(1)}+\ldots+a_{N-1}^{(i)}\hat{a}_{N-1}^{(1)}](1-\hat{\tau}_i/T_c)+[a_0^{(i)}\hat{a}_1^{(1)}+\ldots+a_{N-2}^{(i)}\hat{a}_{N-1}^{(1)}]\hat{\tau}_i/T_c\}\cos\hat{\theta}_i/N \quad ;2 \le i \le K \quad (18)$$

and $\hat{a}^{(1)}$ corresponds to a fixed choice for the values of the components of the random vector $a^{(1)}=(a_0^{(1)}a_1^{(1)}\ldots a_{N-1}^{(1)})$.

From the total probability formula and the fact that n_0^* and $I_i^0(\hat{\tau}_i,\hat{\theta}_i)(2 \le i \le K)$ are symmetric random variables, we conclude that

$$p=\sum_{a^{(1)}=\hat{a}^{(1)}} \Pr(Y \ge 0)\Pr(a^{(1)}=\hat{a}^{(1)}) \quad (19)$$

all possible choices of $\hat{a}^{(1)}$

We now present two lemmas, which will help us define the upper bound p_u

Lemma 1

$$\Pr(Y \ge 0) \le \exp(-z)E[\exp(zn_0^*)] \prod_{i=2}^{K} E\{\exp[z(P_i/P_1)^{1/2}\hat{I}_i^0(\hat{\tau}_i,\hat{\theta}_i)]\} \quad ;z \ge 0 \quad (20)$$

where E denotes the expectation operator.

Lemma 1 is a consequence of the well known Chernoff bound and the fact that the random variables $\hat{n}_0^*$ and $\hat{I}_i^0(\hat{\tau}_i,\hat{\theta}_i)$ $(2 \le i \le K)$ are independent. A proof of Chernoff's bound can be found in [6].

Lemma 2

$$E\{\exp[z(P_i/P_2)_1^{1/2}\hat{I}_i(\hat{\tau}_i,\hat{\theta}_i)]\}$$
$$\leq E\{\exp[z(P_i/P_1)^{1/2}\hat{I}_i(0,0)]\} \qquad (21)$$
$$;2\leq i\leq K,\quad z\geq 0$$

Lemma 2 is proven [7]. Its proof is based on the increasing nature of the function $s(t)=e^t+e^{-t}$ $(t\geq 0)$ [i.e. as t increases $(t\geq 0)$, $s(t)$ increases too].

From lemmas 1 and 2 we take

$$\Pr(Y\geq 0)\leq \exp(-z)E[\exp(zn_0^*)]$$
$$\prod_{i=2}^{K} E\{\exp[z(P_i/P_1)^{1/2}I_i^0(0,0)]\}\ ;z\geq 0 \qquad (22)$$

From formulas (18) and (22) we see that the upper bound on $\Pr(Y\geq 0)$ does not depend on the specific choice of $\hat{a}^{(1)}$. As a result (see(19)),

$$p\leq \exp(-z)E[\exp(zn_0^*)]$$
$$\prod_{i=2}^{K} E[\exp[z(P_i/P_1)^{1/2}J_i]\} \qquad ;z\geq 0 \quad (23)$$

where

$$J_i=[\sum_{j=0}^{N-1} a_j^{(i)}]/N \quad ;2\leq i\leq K \quad (24)$$

Let us finally denote

$$v(z)=\exp(-z)E[\exp(zn_0^*)]$$
$$\prod_{i=2}^{K} E\{\exp[z(P_i/P_1)^{1/2}J_i]\} \qquad :z\geq 0 \ (25)$$

and z^* the value of z, which minimizes $v(z)$. We define p_u as follows:

$$p_u=v(z^*) \qquad (26)$$

It is then obviously true that

$$p\leq p_u \qquad (27)$$

(12), (13), (27), the independence of p_u from $\hat{\underline{\tau}}$ and $\hat{\underline{\theta}}$ and the increasing nature of g(p) allow us to write

$$P_e(K)\leq \sum_{i=e+1}^{M} \begin{bmatrix} M \\ i \end{bmatrix} p_u^i(1-p_u)^{M-i} \qquad (28)$$

(28) provides us with an upper bound on the packet error probability $P_e(K)$. We denote this upper bound $P_e^u(K)$.

3.b Numerical results.

In table 1, we present the upper bound $P_e^u(K)$ on the packet error probability $P_e(K)$, when all interfering signals (i.e. packets #2,...,#K) are assumed to be 0 dB, 3 dB or 6 dB stronger than the desired transmission (i.e. packet #1). Results are presented for various K values. In table 1, we assume a signal to noise ratio E_b/N_o of 12 dB or 15 dB (note that $E_b=P_1T$), and N=31. In table 2, similar results are shown for the N=127 case. The results in tables 1 and 2 correspond to the (1023,513) BCH code. The entries in tables 1 and 2, which are shown as upper bounds, are loose upper bounds of $P_e^u(K)$, and their only purpose is to indicate the approximate order (in powers of 10) of $P_e^u(K)$.

An important observation, stemming from the results shown in tables 1 and 2, is that the performance of direct sequence spread spectrum packet radio networks deteriorates rapidly for cases of interfering signals which are moderately stronger than the desired transmission (the near-far problem).

4. Conclusions

We presented an upper bound on the packet error probability induced in direct sequence spread spectrum packet radio networks. An important advantage of the bound p_u, derived in section 3 is that its validity does not rely on any assumptions about the joint probability density function $f_{\tau,\theta}$ of all delays and phases (e.g. independence of the delays and phases). The only assumption that we used for the derivation of p_u is that each delay and phase includes in its range the zero value. Furthermore, the form of the bound p_u is simple and easily computable (see (25) and (26)). Once p_u is calculated, the computation of $P_e(K)$ for BCH codes becomes a straightforward task (see formula (28)). More importantly, our presentation in section 3 has shown that the upper bound on the packet error probability is valid independently of whether signals arrive with equal or unequal powers at the receiver site.

In [7] we mention some ways of improving the upper bound of section 3 at the expense of increased computational complexity. We are currently examining the improvement of the bound, as well as, its applicability to other types of forward error correction codes (e.g. convolutional codes).

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Table 1

The upper bound $P_e^u(K)$ on the packet error probability $P_e(K)$.

E_b/N_o=12 dB; N=31

	0 dB		3 dB		6 dB
K	$P_e^u(K)$	K	$P_e^u(K)$	K	$P_e^u(K)$
4	2.32D-11	2	<1.0D-41	2	8.73D-03
5	2.81D-02	3	1.83D-02		

E_b/N_o=15 dB; N=31

	0 dB		3 dB		6dB
K	$P_e^u(K)$	K	$P_e^u(K)$	K	$P_e^u(K)$
3	<1.0D-57	2	<1.0D-57	2	5.81D-07
4	2.62D-14	3	3.88D-06		
5	1.11D-05				

Table 2

The upper bound $P_e^u(K)$ on the packet error probability $P_e(K)$

E_b/N_o=12 dB; N=127

	0 dB		3dB		6 dB
K	$P_e^u(K)$	K	$P_e^u(K)$	K	$P_e^u(K)$
9	<1.0D-40	5	<1.0D-41	3	<1.0D-41
10	<1.0D-30	6	<1.0D-22	4	2.03D-13
11	<1.0D-22	7	3.18D-13	5	9.81D-03
12	<1.0D-16	8	1.88D-06		
13	4.79D-13	9	1.11D-02		
14	2.95D-9				
15	2.54D-06				

E_b/N_o=15 dB; N=127

	0 dB		3 dB		6 dB
K	$P_e^u(K)$	K	$P_e^u(K)$	K	$P_e^u(K)$
10	<1.0D-52	6	<1.0D-40	3	<1.0D-57
11	<1.0D-40	7	<1.0D-22	4	<1.0D-23
12	<1.0D-30	8	3.11D-13	5	1.40D-06
13	<1.0D-22	9	1.85D-06		
14	<1.0D-16	10	1.10D-02		
15	5.11D-13				